Physics 307 Experiment 3:
Probability Distributions and the Decay of Quantum States

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1 Purpose

To use Poisson statistics to analyze decay data of Cesium-137 and to calculate the half-life of excited Barium-137.

2 Theory

The decay process of Cesium-137 is

$$^{137}\mathrm{Cs} \longrightarrow^{137} \mathrm{Ba}^* \longrightarrow^{137} \mathrm{Ba} + \gamma(662 \text{ keV})$$

where the Cesium first decays into an excited state of Barium through Beta decay, then when the excited state relaxes, a 662 keV photon is emitted. Since the arrival of Gamma-rays in our detector is independent of time since the last detection event, this process can be modeled through Poisson statistics. In the first part of this experiment, we are going to be testing the accuracy of this statistical model.

In the second part of the experiment, we will calculate the half-life of the

$$^{137}\mathrm{Ba}^* \longrightarrow^{137} \mathrm{Ba} + \gamma(662 \text{ keV})$$

transition using both a linear and non-linear model.

3 Procedure

3.1 Part 1: Probability Distributions

In the first part of this lab, we wanted to investigate elements Poisson statistics for counting gamma-ray events. We used the same set up at experiment 1: a Cesium-137 source placed below a Sodium Iodide (NaI) crystal, after which a Photomultiplier Tube (PMT) would pickup incident gamma-radiation and send a count signal to a Multichannel Analyzer (MCA).

For our first trial we set up the MCA to count for 200ms per channel. There are 1024 channels on the MCA, so the total duration of the trial was 204.8 seconds. After gathering the data, we imported it into Matlab for processing. Figure 1 shows the histogram of the data, using a bin size of 5. Since our goal is to verify Poisson statistics, we first computed the mean and standard deviation directly:

Mean (μ): 179.88
St Dev (σ): 13.82

And compared these values with those predicted by Poisson statistics:

Poisson St Dev : 13.41

and calculated the uncertainty in the mean:
Mean Uncertainty ($\sigma_{\bar{x}}$) = 5.62

Lastly, we selected 9-10 random data points and computed the the Poisson uncertainty $\sigma^{\text{Poisson}} = \sqrt{x}$ for each one. We compared the data points to the true mean found above. These comparisons are shown in the Appendix. We then calculated the standard deviation of the mean and compared with the true mean.

$\mu^{\text{Poisson}} = 179.6$
μ^{Poisson} Uncertainty: 13.4
Poisson uncertainty std of mean 0.42

The Poisson mean is well within our uncertainty. All the values match within their errors, so Poisson statistics is a valid method of analyzing this process.

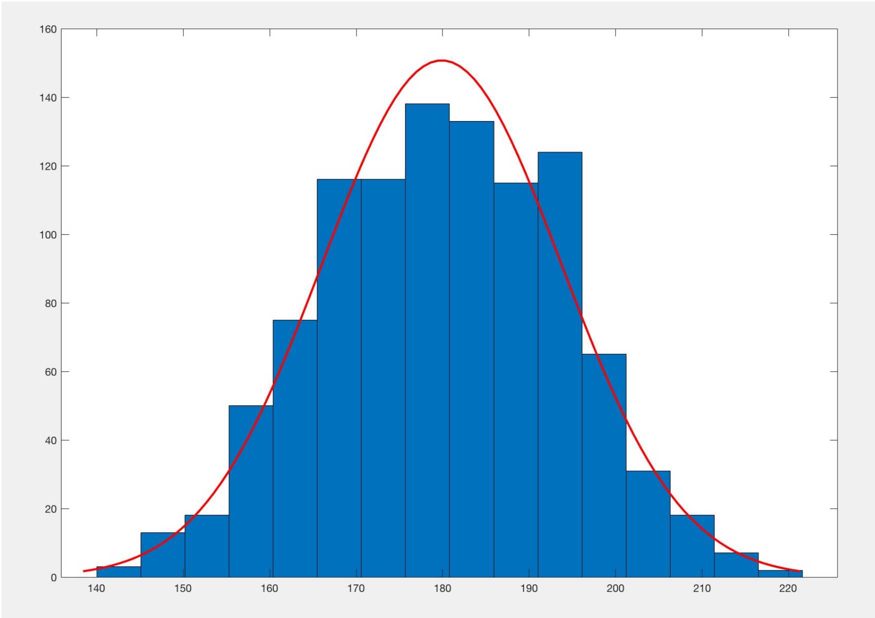

Figure 1: A histogram and Gaussian fit of our first (200ms dwell time) trial

Next we repeated the above calculations using an 800ms dwell time:
Mean (μ): 718.3
St Dev (σ): 26.36

Poisson St Dev ($\sigma_{\bar{x}}$): 26.80
Mean Uncertainty ($\sigma_{\bar{x}}$) = 22.45

And the values calculated from our 9-10 values were:
$\mu^{\text{Poisson}} = 718.1$
μ^{Poisson} Uncertainty: 26.8
Poisson uncertainty std of mean 0.84

Again, all the values match up very well, so Poisson statistics is a valid method of analyzing this process.

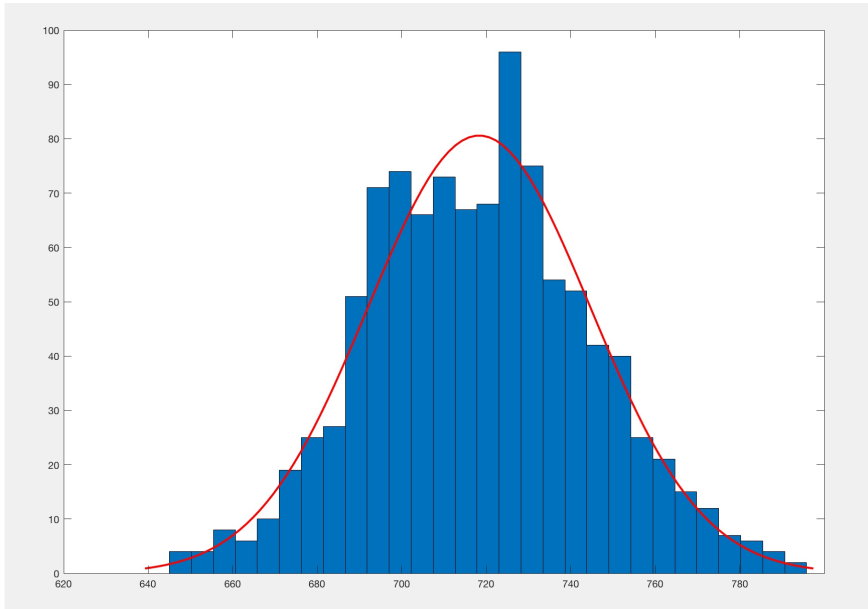

Figure 2: A histogram and Gaussian fit of our second (800ms dwell time) trial

3.2 The Decay of Excited Quantum States

In the second part of this lab, we wanted to measure and analyze the decay process of the excited state of Barium. We first had to prepare our sample by eluting the Cs-137 source. This consists of flushing a concentrated NaCl solution through the source to remove some of the Ba-137 while leaving the Cesium in place. After the eluting process, we then had a small beaker of excited Barium dissolved in the NaCl solution. We placed this sample below the photomultiplier tube and used the multichannel analyzer to measure its decay process.

The half-life of excited Barium is just a few minutes, but we gathered data for approximately one hour to ensure we captured the entire decay process. We set the dwell time to 10 seconds per channel. Once the graph flat-lined, we knew we could stop the data collection since we had reached the background radiation level.

Barium follows the equation for exponential decay $N(t) = N_0 e^{-\lambda t}$, where λ is the decay rate, N_0 is the initial number of gamma-ray counts, and the lifetime τ is equal to $1/\lambda$. We uses both a non-linear and linear fit to calculate the half-life of excited Barium. From the non-linear fit

$$y = Ae^{-t/\tau} + B$$

we can read off the parameters $N_0 = A$, τ, and B is the ambient background radiation. Figure 4 shows our non-linear fit for excited Barium decay. Our non-linear fit model was

f(x) = a*exp(-b*x)+c
Coefficients (with 95% confidence bounds):
a = 1.492e+04 (1.49e+04, 1.494e+04)
b = 0.004519 (0.004509, 0.00453)
c = 138.6 (135.3, 141.9)

Goodness of fit:
SSE: 2.679e+05
R-square: 0.9999
Adjusted R-square: 0.9999

RMSE: 27.55

So our background is 139 counts every 10 seconds, our initial counts is 14,920, and lambda is 0.004519.

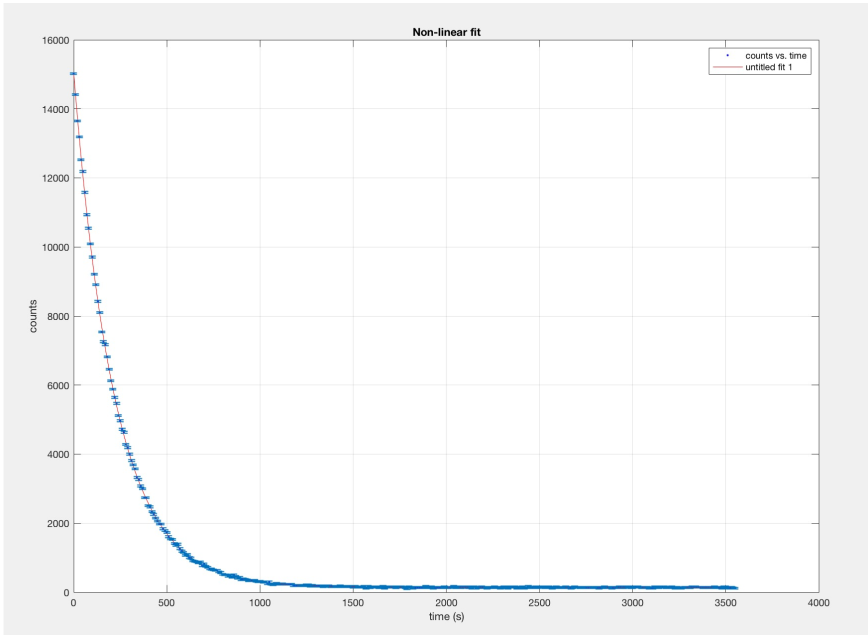

Figure 3: A non-linear fit for our decay data

Alternatively, we could also plot the data on a log plot to find the coefficients.

$$N(t) = N_0 e^{-\lambda t} + B$$
$$\ln\left(\frac{N(t) - B}{N_0}\right) = -\lambda t$$

the slope of the linear fit is then $-\lambda$, and we can exclude the background since this will show up clearly on a log plot. Figure 4 shows this linear fit. The linear fit coefficients were

f(x) = m*x + b
Coefficients (with 95% confidence bounds):
m = -0.004479 (-0.004525, -0.004434)
b = -0.02629 (-0.06156, 0.00898)

Goodness of fit:
SSE: 1.444
R-square: 0.9965
Adjusted R-square: 0.9965
RMSE: 0.1042

So lambda is 0.004479.

Lastly, we used our values of lambda to calculate the half-life

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \tau \ln(2)$$

Our calculated half-life values are given in the following results section.

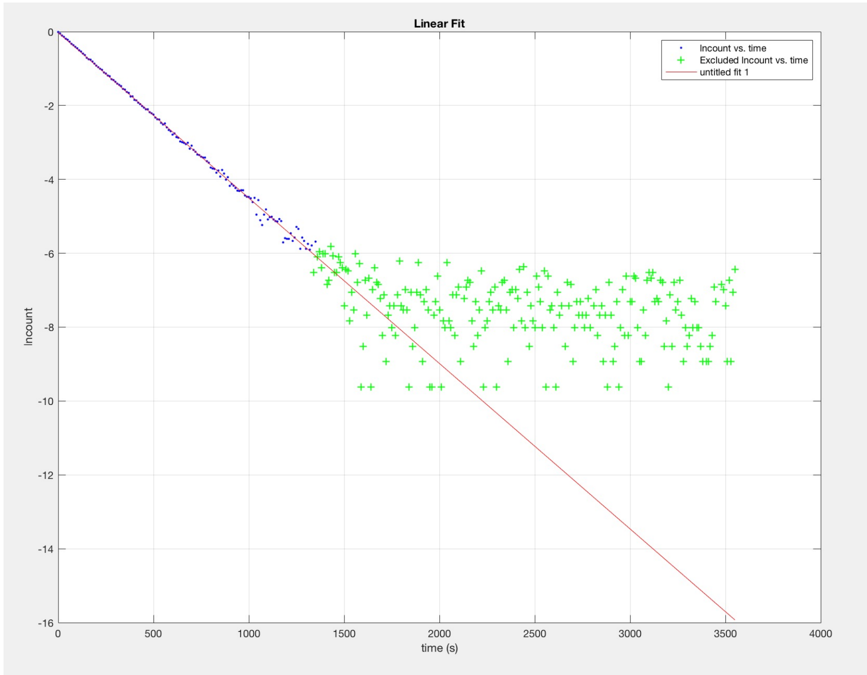

Figure 4: A linear fit for our decay data

In order to calculate the error in our measurement, we had to find

$$\Delta y = \Delta \ln\left(\frac{N(t) - B}{N_0}\right)$$

We let

$$z = \frac{N(t) - B}{N_0}$$

so we can calculate the error of y in terms of z.

$$y = \ln(z)$$

$$\Delta y = \frac{\partial}{\partial z}\Big(\ln(z)\Big)\Delta z = \frac{\Delta z}{z}$$

Define the numerator of z as

$$n = T(t) - B$$

and the denominator as

$$d = N(0).$$

We know from simple error propagation that

$$\Delta n = \sqrt{\Delta T^2 + \Delta B^2}$$

$$\Delta d = \sqrt{N(0)} \quad \text{(from Poisson statistics)}$$

$$\frac{\Delta z}{z} = \sqrt{\Big(\frac{\Delta n}{n}\Big)^2 + \Big(\frac{\Delta d}{d}\Big)^2}$$

So finally

$$\Delta z = z\sqrt{\frac{\Delta T^2 + \Delta B^2}{(T(t) - B)^2} + \frac{1}{N(0)}}$$

We estimated ΔT as the range of background radiation fluctuations, which was 25 counts per 10 seconds. Our error in the average background was 1. Our error in N(0) is the same as the error of T(0). Using these values we added error bars to figure 4, which can be seen in figure 5.

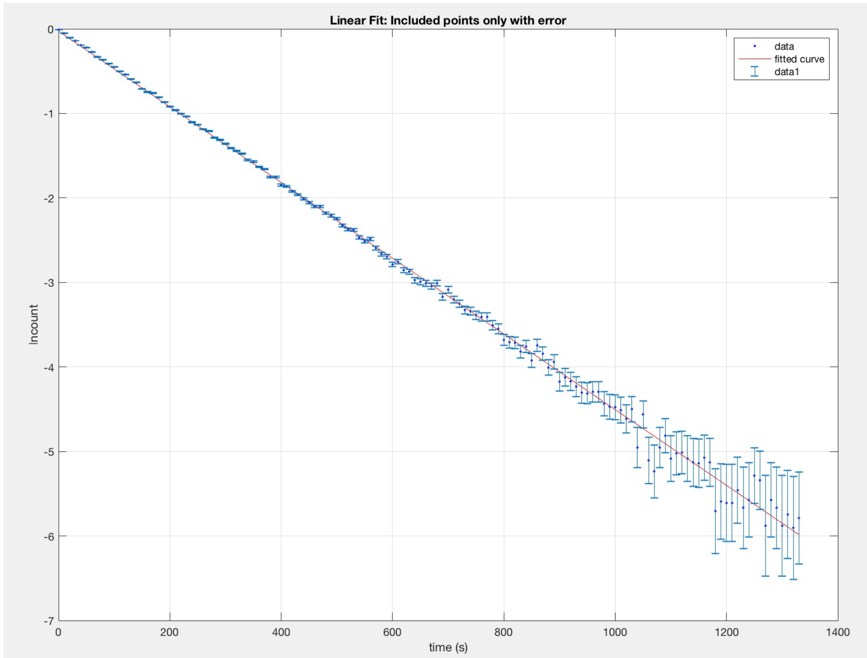

Figure 5: A linear fit for our decay data, ignoring the excluded values, with error bars

4 Results

Non-linear λ (1/s)	0.004519 $\pm$ 0.000011
Non-linear $t_{1/2}$ (s)	153.35 $\pm$ 0.37
Linear λ (1/s)	0.004479 $\pm$ 0.000045
Linear $t_{1/2}$ (s)	154.7 $\pm$ 1.55
Actual $t_{1/2}$ (s)	153.12 $\pm$ 0.06

Table 1: Our calculated half-life results, compared with the accepted value

Table 1 shows our results. Our calculated values for the half life of excited Barium matches to within 1% of the accepted value. The accepted value lies within our error bounds for the non-linear trial, but it is just outside of our error bound for the linear trial. This was likely caused by using only the background radiation as a source of error while ignoring other sources like instrument accuracy and computer dead time. Overall, our measurements matched the accepted values very well.

5 Post-Questions

1. What is $N(t)$ when $t = \tau$?

$$N(t) = N(0)e^{-t/\tau}$$
$$N(\tau) = N(0)e^{-1}$$
$$N(\tau) = 0.37\ N(0)$$

2. Explain why the observed $t_{1/2}$, of the 662 keV gamma-rays from the Cs^{137} source is 33 years, even though the γ-rays you are counting come from the decay of the Ba^{137}, which has a $t_{1/2}$, of only a few minutes.

   The half life of the gamma-rays represent the half-life of the entire decay process. Since Cesium-137 does not emit gamma-rays on its own (it must decay into an excited Barium-137 first), and the half life of Cs-137 is much greater than the half life of excited Ba-137, the emission of gamma rays decay at a rate similar to the Cesium.

3. Is the time interval between when the source is eluted and when you begin your measurements critical? Why or why not?

   It is critical in the sense that as the Barium decays, the ratio of background noise in the data will be greater. However, as long as this duration is not too long, the exponential decay will still be visible, and all the parameters of interest can be calculated. The error will be just be higher.

4. What is dead-time? Is the computer channel dead-time of 1.8 μs significant?

   Dead time is the shortest time that the computer can register two counts. To determine if this time is significant, calculate the maximum average time in between counts for the data set:

$$\frac{\text{Dwell Time}}{\text{Counts}} = \frac{10\text{s}}{14{,}411} = 6.9 \times 10^{-4}\ \text{sec/event}$$

   which means that one gamma ray is counted every 300 times the dead time of 1.8μs. The dead time is not significant.

5. In another experiment the following data set is obtained (shown on next page). When analyzed one student group claims that a single exponential decay is adequate while another group states that to fully explain this data a minimum of two decay constants are needed. Whose assertion do you support and explain how you've arrived at this conclusion. It may help to include a figure or two of your analysis. **In both cases assume the background level is zero.**

   In order to answer this question, I plotted the decay process on a log plot. There is a clear linear relationship in the first part of the graph, however this does not match the second part of the data (see figure 6). From this, I conclude that there is a secondary process during the later half of the data (see figure 7). Although weaker than the first process, it is clearly not background since it still has some non-zero decay slope. Also, it can't be related to first process since they have different slopes.

6 Conclusion

Poisson statistics is a useful and accurate method of analyzing sequences of unrelated events. In the limit as the number of events gets large, a Poisson-statistics process approximates a Gaussian. In this experiments we used two methods for calculating the half-life of an excited state of Barium-137: linear and non-linear fits. Both methods resulted in a half life that was within 1% of the accepted values.

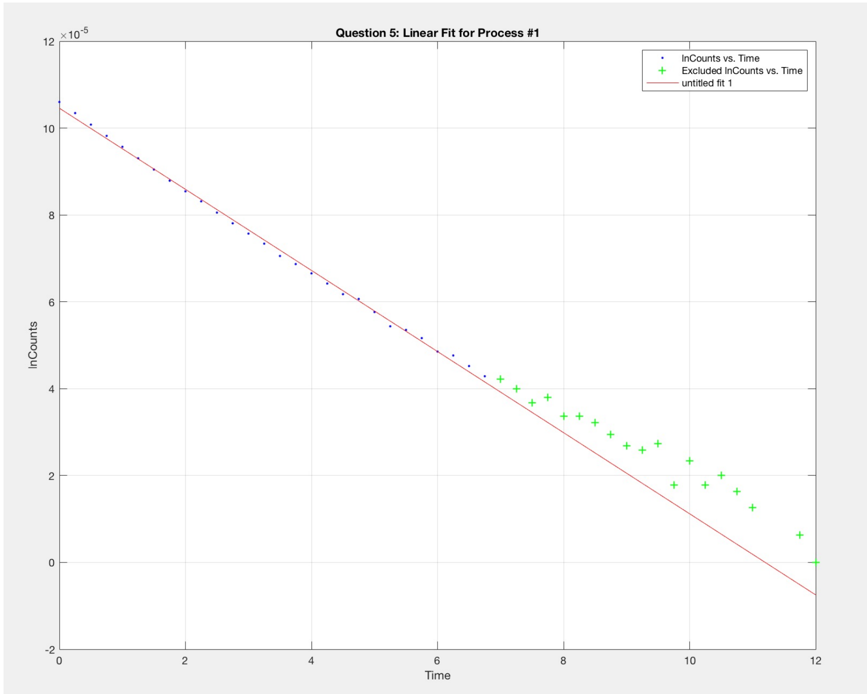

Figure 6: A linear fit for process #1

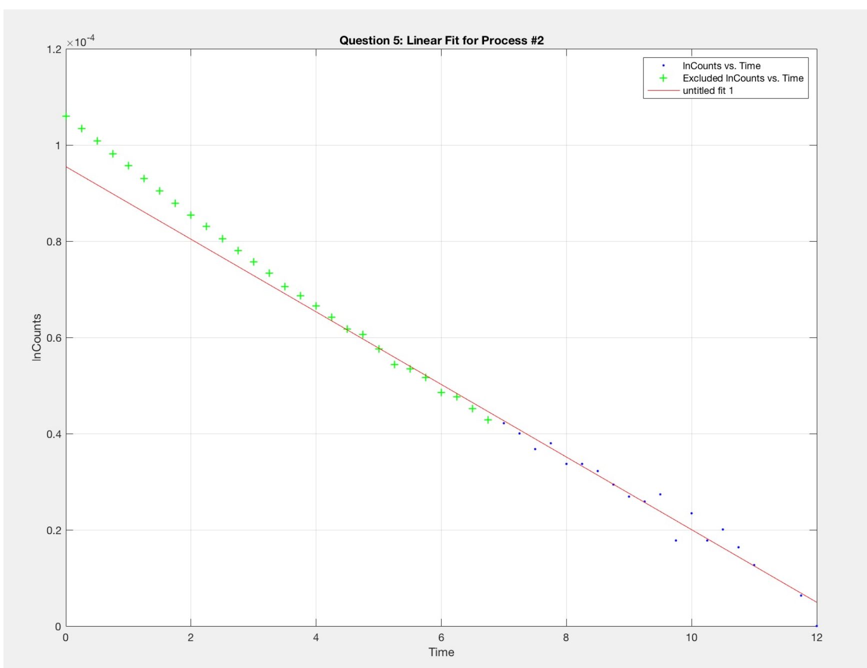

Figure 7: A linear fit for process #2

7 Appendix

200ms Trial

Channel	Counts	Poisson Uncertainty	Counts minus mean	Comparing uncertainties
3	166	12.9	-13.88	-1
11	205	14.3	25.12	-10.8
19	178	13.3	-1.88	11.5
128	179	13.4	-0.88	12.5
134	148	12.2	-31.88	-19.7
140	157	12.5	-22.88	-10.4
306	172	13.1	-7.88	5.2
382	167	12.9	-12.88	0
390	170	13	-9.88	3.2
424	193	13.9	13.12	0.8

800ms Trial

Channel	Counts	Poisson Uncertainty	Counts minus mean	Comparing uncertainties
3	731	27	12.7	14.3
4	734	27.1	15.7	11.4
26	713	26.7	-5.3	21.4
27	742	27.2	23.7	3.5
55	670	25.9	-48.3	-22.4
158	730	27	11.7	15.3
257	707	26.6	-11.3	15.3
341	695	26.4	-23.3	3.1
412	723	26.9	4.7	22.2
470	701	26.5	-17.3	9.2