Physics 307 Experiment 4: Cavendish Measurement of Big G

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1 Purpose

To replicate Henry Cavendish's measurement of Newton's gravitational constant using the damped harmonic oscillations of a torsion pendulum.

2 Theory

The universal gravitational constant G remains to this day one of the most difficult properties of our universe to measure. Much of this is due to the fact that the gravitational force is so weak compared to the other fundamental forces, and we are confined to Earth's gravitational field, which can easily overpower any precise measurements we attempt to make. In this experiment, we used a torsion pendulum, which is effectively isolated from the effects of Earth's gravity to measure the gravitational attraction between lead masses.

When the lead masses are moved near the pendulum, the gravitational force sets it into simple harmonic motion. The wire supplies a restoring force, which is proportional to the torsion constant K. Based on the frequency of oscillations, we can work out what the force is between the masses, and with it, the gravitational constant. The gravitational force is

$$F = G\frac{m_1 m_2}{r^2}$$

which results in a torque

$$\tau = 2\frac{GMm}{d^2}\frac{L}{2}.$$

The gravitational torque defines a new equilibrium angle where the torque from gravity is equal to the restoring torque from the torsion constant of the wire. Here θ_0 denotes the equilibrium angle.

$$2\frac{GMm}{d^2}\frac{L}{2} = K\theta_0$$

$$G = \frac{K\theta_0 d^2}{MmL}$$

$$G = \frac{2m\frac{L}{2}^2\omega_0^2\theta_0 d^2}{MmL}$$

$$G = \frac{L\omega_0^2\theta_0 d^2}{2M}$$

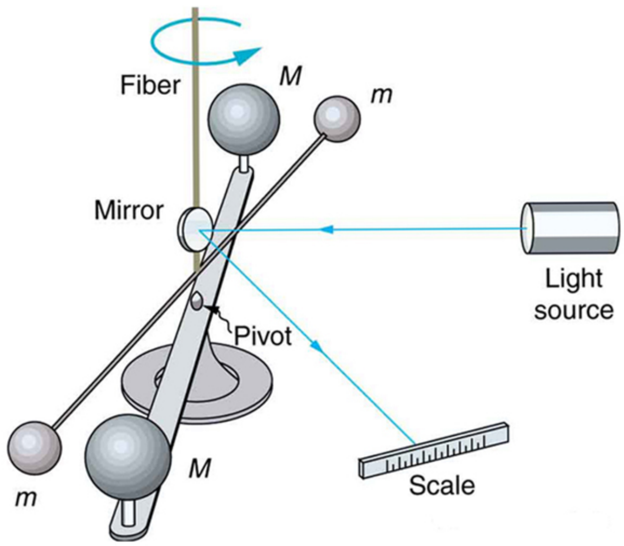

Figure 1: The setup of the experiment

Source: https://nigerianscholars.com/tutorials/uniform-circular-motion-gravitation/the-cavendish-experiment-then-and-now/

3 Procedure

Figure 1 shows the experimental setup. Two small lead masses are suspended on a metal bar, which has a mirror attached to the center. A laser is positioned so that it hits the center mirror and is reflected on to opposite wall. We can then use the position of the reflected laser beam to determine the rotation angle θ of the pendulum. We recorded the position of the laser once every thirty seconds. Since we expected one period of oscillation to take approximately ten minutes, this would give us sufficient data points to determine the frequency and other characteristics of the oscillations.

There are essentially only three values that we had to measure to determine the gravitational constant, ω_0, θ_0, and M. L and d were unique to the pendulum we were using and were given in the lab description. In order to determine ω_0 and θ_0, we had to complete two trials, one with the the masses positioned on each side of the pendulum. We used Microsoft Excel for the initial data collection, but once we collected data from 4-5 periods, we imported it into MatLab for processing.

Position data vs. time: Trial 1

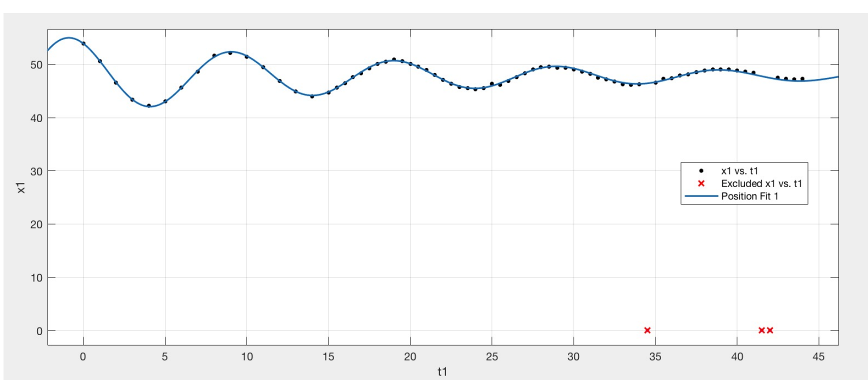

Figure 2: The position curve fit for our first data trial.

Position data vs. time: Trial 2

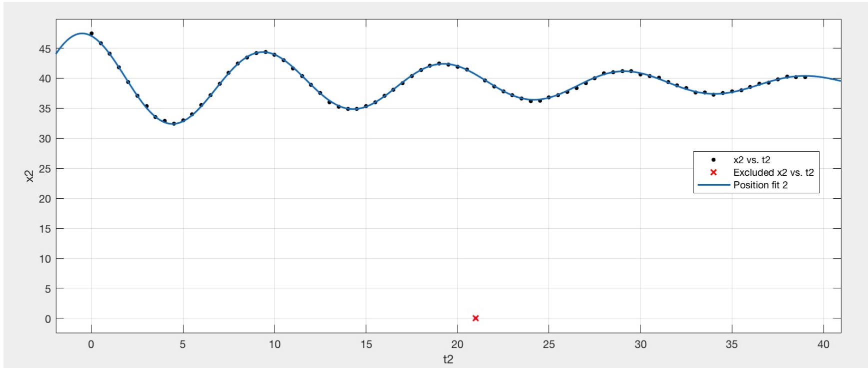

Figure 3: The position curve fit for our second data trial.

We used MatLab's cftool in order to fit a curve to our data. Since the torsion pendulum behaves as a damped harmonic oscillator, we used the equation $x(t) = A^{(-bt)}\cos(\omega t + \phi_0) + x_0$ to fit the data appropriately. We first fit a curve to our position data in order to determine the correct equilibrium position (x_0) value. Figures 2 and 3 show this curve fit.

Once we determined the equilibrium value, we could convert all of our position data to displacement angle using the sin relationship between displacement position and distance to the wall (D), which we measured to be 3.49 ± 0.005 m. Note that here the 2θ is necessary because the angle is doubled due to the laser being reflected.

$$\sin(2\theta) = 2\theta = \frac{x - x_0}{D}$$
$$\theta = \frac{x - x_0}{2D}$$

We then fit a curve to our angle data. The curve fit images and equation models are provided on the following page. Cftool determined that the frequency of our first trial was $\omega_0 = -0.6323\frac{\text{rad}}{\text{s}}$ and the frequency of our second trial was $\omega_0 = -0.6362\frac{\text{rad}}{\text{s}}$.

Angle data vs. time: Trial 1

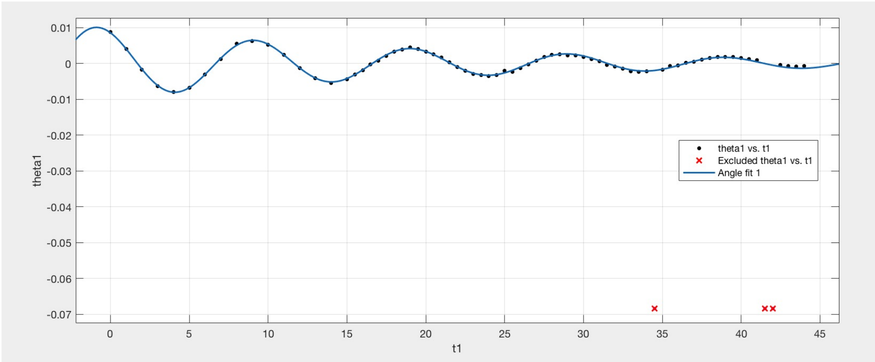

Figure 4: The angle curve fit for our first data trial.

General model (trial 1):
f(x) = a*exp(-b*x)*cos(o*x+p)+c
R-square: 0.9936
Coefficients (with 95 confidence bounds):
a = 0.009691 (0.009387, 0.009995)
b = 0.04471 (0.0428, 0.04661)
c = 4.701e-06 (-5.536e-05, 6.476e-05)
o = -0.6323 (-0.6342, -0.6304)
p = -0.4898 (-0.5204, -0.4593)

Angle data vs. time: Trial 2

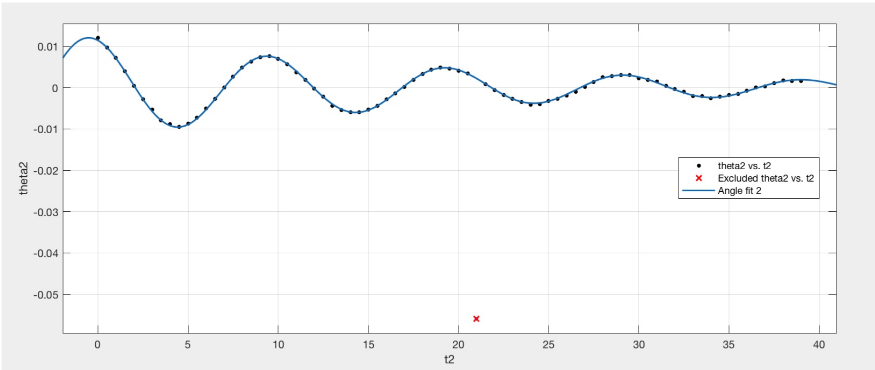

Figure 5: The angle curve fit for our second data trial.

General model (trial 2):
f(x) = a*exp(-b*x)*cos(o*x+p)+c
R-square: 0.9979
Coefficients (with 95 confidence bounds):
a = 0.01179 (0.0116, 0.01197)
b = 0.04676 (0.04552, 0.04799)
c = -3.982e-06 (-5.1e-05, 4.304e-05)
o = 0.6362 (0.6349, 0.6375)
p = 0.2543 (0.2381, 0.2704)

4 Results

	Trial 1	Trial 2
$\omega_0(\frac{\text{rad}}{\text{s}})$	-0.6323	-0.6362
$\Delta\omega_0(\frac{\text{rad}}{\text{s}})$	0.013	0.013
θ_0 (rad)	0.006246	-0.006246
$\Delta\theta_0$ (rad)	0.0000064	0.0000064
M (kg)	1.50318	1.50318
ΔM (kg)	0.00007	0.00007

From our results, we calculated the gravitational constant using the derived formula (see "theory" section).

$$G_{trial1} = 5.123 \times 10^{-11} \frac{m^3}{kg\ s^2}$$
$$G_{trial2} = 5.188 \times 10^{-11} \frac{m^3}{kg\ s^2}$$

The formula for G has an associated error propagation of

$$\frac{\Delta G}{G} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + 4\left(\frac{\Delta\omega_0}{\omega_0}\right)^2 + \left(\frac{\Delta\theta_0}{\theta_0}\right)^2 + 4\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta M}{M}\right)^2}$$

We chose 5 mm as our value for Δd since the lead masses were positioned by hand. Using the above experimental uncertainties as well as the provided $\Delta L = 0.2$ cm, our uncertainty in G was

$$\Delta G = 1.173 \times 10^{-11} \frac{m^3}{kg\ s^2}$$

This puts the true value of G (6.674) just past the upper range of our uncertainty (6.296). This discrepancy is likely due to our values for ω_0 and d. Because the period of oscillations was so slow, it was difficult to accurately determine the when the pendulum had completed a full oscillation. d was also difficult to determine since we had to be cautious not to disturb the pendulum while positioning the masses.

5 Conclusion

The Cavendish experiment is a powerful method for determining the universal gravitational constant, isolated from Earth's gravity. We were able to determine the gravitational constant to within 23% of the currently accepted value. As a part of this experiment, we analyzed the error in our value for G, and found that the true value (6.674) existed just beyond the upper range of our uncertainty (6.296). A more accurate value for G could likely be found by modifying how we determine the distance to the lead masses (d), which was most likely our largest source of systematic error.