Physics 307 Experiment 5:
Attenuation of Gamma-rays in Matter

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Note: Answers to lab **Questions** are highlighted in bold-face

1 Purpose

To demonstrate the exponential decay of gamma-rays in lead shielding, to compare the shielding effectiveness of lead vs. Aluminum, and to measure the decay coefficient κ for Aluminum.

2 Theory

Gamma-rays decay in matter according to the following formula

$$I = I_0 e^{-\kappa x}$$

where I is the intensity, κ is the decay coefficient and x is the thickness of the barrier. In this experiment, we are going to be testing this relationship and measuring the value of κ for Aluminum.

3 Procedure

3.1 The Exponential Dependence of Transmittance

In the first part of this experiment, we wanted to measure how gamma-ray transmittance varies with shielding thickness. We used a set of lead plates, and placed one plate in between the Cesium-137 source and the NaI crystal detector. We then used the multi-channel analyzer (MCA) to measure the time it took to register 10000 counts. The intensity is then given by

$$I = \frac{10000 \text{ counts}}{t}$$

We measured the thickness of each lead plate using the micrometer. Then we plotted the intensity of gamma-rays versus the shielding thickness. The expected intensity is also shown, which follows the following formula:

$$I = I_0 e^{-\kappa x}$$

where kappa is 1.18 cm^{-1} for lead and I_0 is the measured zero-shielding intensity. We fit an exponential curve to our measured values to determine how well the process followed our expectation. The exponential model was

General model Exp1:
f(x) = a*exp(b*x)
Coefficients (with 95% confidence bounds):

a = 1659 (1624, 1694)
b = -0.1224 (-0.1325, -0.1124)

Goodness of fit:
SSE: 679.1
R-square: 0.9985
Adjusted R-square: 0.998
RMSE: 15.05

With an R-square value of 0.9985, it's safe to say that the process is indeed exponential, and we also get a reasonable value for κ_{pb} of 0.1224 cm^{-1}.

Measured Width (mm)	Actual width (mm)	time (s)	Intensity (Counts/sec)	Expected (Counts/sec)
0	0	6.01 ± 0.01	1664 ± 2.8	1664 ± 2.8
1.45	0.78 ± 0.01	6.7 ± 0.01	1493 ± 2.2	1402 ± 28.3
2.07	1.4 ± 0.01	7.07 ± 0.01	1414 ± 2.0	1303 ± 26.3
3.53	2.86 ± 0.01	8.64 ± 0.01	1157 ± 1.3	1097 ± 22.1
6.75	6.08 ± 0.01	12.61 ± 0.01	793 ± 0.6	750 ± 15.1

Table 1: Our thickness vs. intensity data for the lead barriers

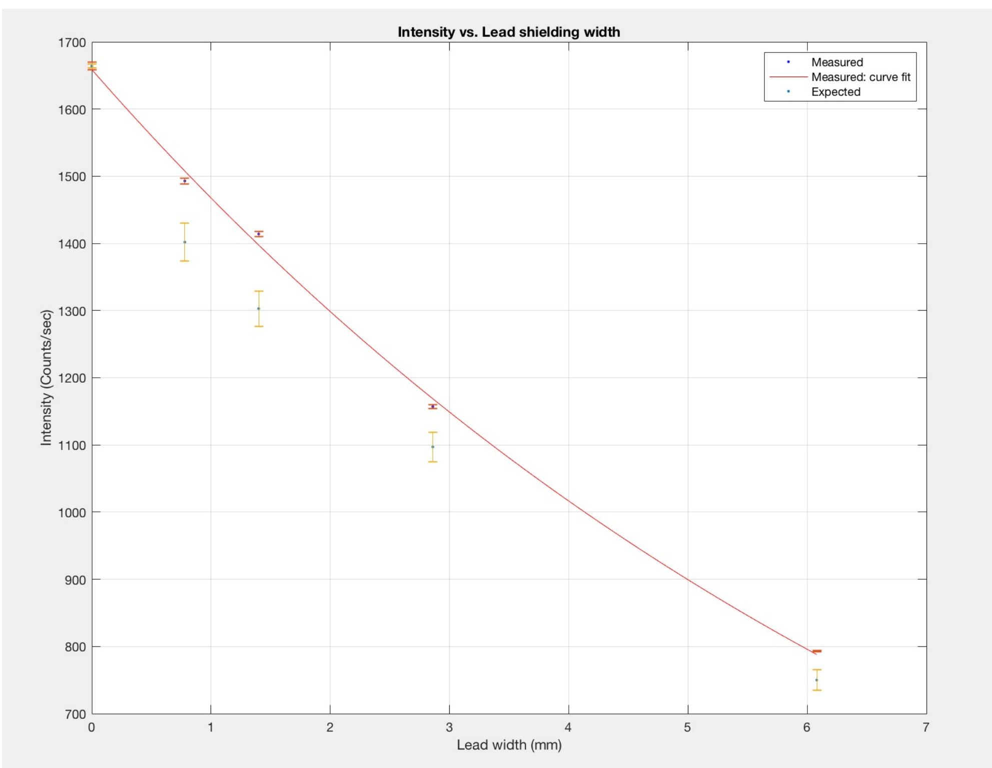

Figure 1: Our measured intensity vs. thickness for the lead barriers

3.2 The Transmittance of Al vs Pb with similar Mass per unit Area

In the second part of the experiment, we tested the effectiveness of Lead vs. Aluminum shielding for the same mass per unit area. Since the lead plates have a much higher mass per area, we had to use multiple Aluminum plates to achieve the same mass per area. We built the following combinations to approximately match the lead plates we had available:

1167: 522+645
2115: 840+522+425+328
3415: 840+655+645+522+425+328

We followed the same procedure as part one, placing the shielding in between the source and detector for each test and measuring how long it took to reach 10000 counts. Table 2 shows our results. Although having a similar mass per area is able to get the values close, Aluminum still lets about 3.5% more radiation through for every 1 g/cm^2 of mass per area. This effect only gets compounded as you use thicker shielding.

Radiation shielding is often given in units of mass per area. This makes sense because you can get a ballpark estimate for how effective the shielding will be without calculating transmission coefficients of each material. Although each material will block radiation by different amounts, mass per area is an important measurement if you want a quick estimate of how effective a given shielding will be.

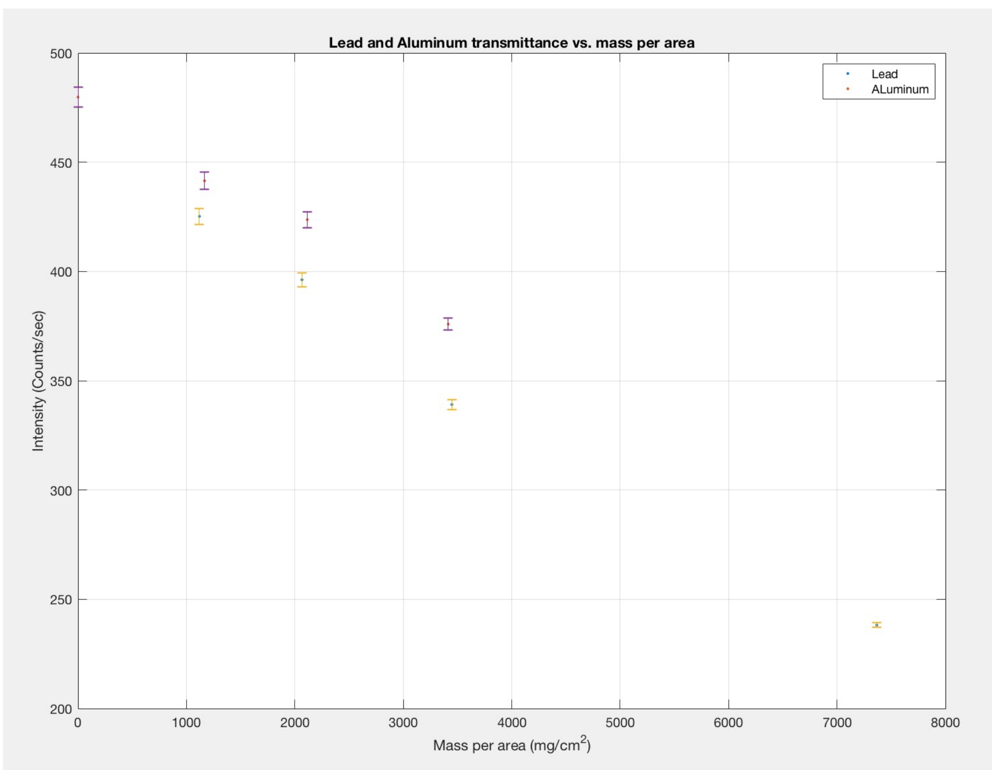

Figure 2: Our measured intensity vs. mass per area for Lead and Aluminum

Lead mass/area	Lead time	Aluminum mass/area	Aluminum time	% difference
0	20.84 ± 0.1	0	20.84 ± 0.1	
1120 ± 5	23.52 ± 0.1	1167 ± 5	22.65 ± 0.1	3.7%
2066 ± 5	25.24 ± 0.1	2115 ± 5	23.6 ± 0.1	6.5%
3448 ± 5	29.49 ± 0.1	3415 ± 5	26.6 ± 0.1	9.8%
7367 ± 5	41.99 ± 0.1	—-	—-	—-

Table 2: Our lead vs. Aluminum acquire times

3.3 Precision Measurement of κ for 662 keV γ-rays in Al

Lastly, we wanted to measure the attenuation coefficient κ for Aluminum. We set up the MCA detector horizontally on the bench and took measurements of the gamma-ray intensity for different thicknesses of Aluminum.

Al (cm)	Z–Unattenuated (Cts./s)	Y–Attenuated (Cts./s)	B–Background (Cts./s)	T
0	66.71 ± 0.0089	66.71 ± 0.0089	2.69 ± 0.00013	1.00 ± 0.000
1	66.71 ± 0.0089	56.56 ± 0.0064	2.69 ± 0.00013	0.84 ± 0.163
3	66.71 ± 0.0089	38.48 ± 0.0030	2.69 ± 0.00013	0.56 ± 0.124
5	66.71 ± 0.0089	27.12 ± 0.0015	2.69 ± 0.00013	0.38 ± 0.099
8	66.71 ± 0.0089	15.82 ± 0.0005	2.69 ± 0.00013	0.21 ± 0.072
9	66.71 ± 0.0089	13.60 ± 0.0004	2.69 ± 0.00013	0.17 ± 0.067
11	66.71 ± 0.0089	9.61 ± 0.0002	2.69 ± 0.00013	0.11 ± 0.057
13	66.71 ± 0.0089	7.09 ± 0.0001	2.69 ± 0.00013	0.07 ± 0.050
15	66.71 ± 0.0089	5.46 ± 0.0002	2.69 ± 0.00013	0.0 ± 0.045

Table 3: Our measured intensity attentuations for differenct thicknesses of Aluminum

In our first method, we plotted ln(T) vs. Aluminum thickness. We then used a linear fit to extract κ, which would show up as the slope due to the equation

$$T(x_i) = \frac{Y_i - B_i}{Z_i - B_i} = e^{-\kappa x_i}$$

Figure 3 shows our linear fit. Our linear fit model was

Linear model:
f(x) = m*x + b
Coefficients (with 95% confidence bounds):
m = -0.1743 (-0.1791, -0.1695)
b = -0.01378 (-0.05032, 0.02276)

Goodness of fit:
SSE: 0.003588
R-square: 0.9993
Adjusted R-square: 0.9991
RMSE: 0.02445

Reading off the slope, we found that κ is 0.1743 $cm^{-}1$.

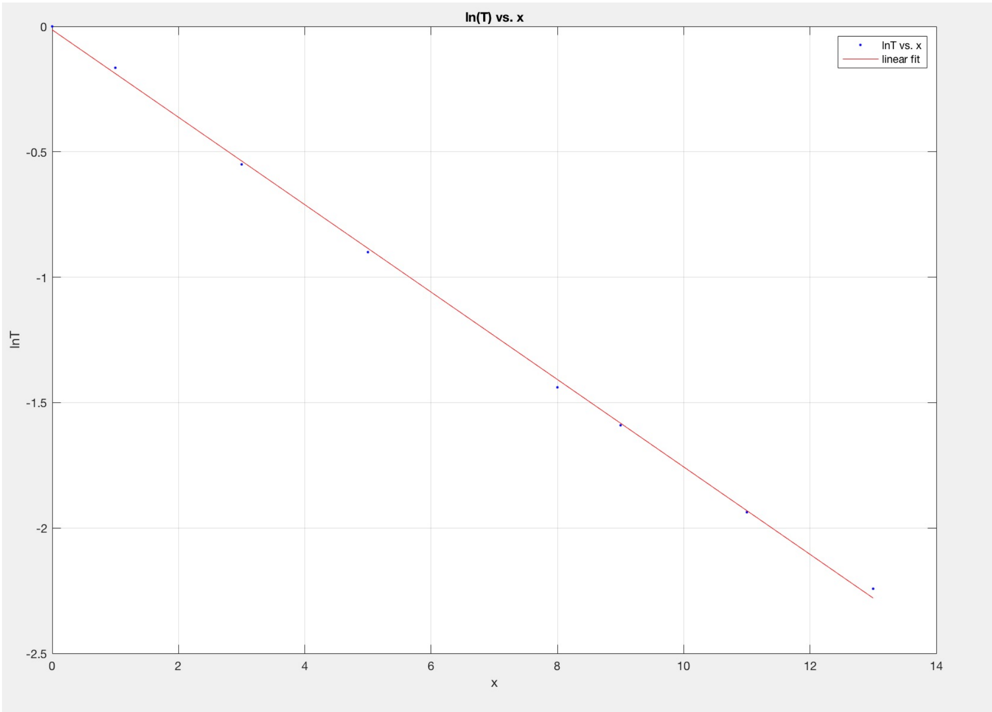

Figure 3: Our linear fit for transmission vs thickness

This is significantly lower than the known value of 0.201 cm$^-1$, so we used an alternative method of calculation using the weighted average. The weighted average should take better account of the errors involved in measurement, so we hope to get a more accurate value for kappa. For each T value in table 3, we calculated κ using $T(x_i) = e^{-\kappa x_i}$. We then had to calculate the uncertainty in each kappa value in order to determine the weighting. The error in κ follows the following calculation:

$$\kappa_i = \ln\left(\frac{y-B}{z-B}\right)/l_i$$

Since y, z, and B follow Poisson statistics,

$$\Delta y = \sqrt{y}\ ,\ \Delta z = \sqrt{z}\ ,\ \Delta B = \sqrt{B}$$

We let

$$A = y - B\ ,\ C = y - B$$

So

$$\Delta A = \sqrt{\Delta y^2 + \Delta B^2} = \sqrt{y+B}$$
$$\Delta C = \sqrt{\Delta z^2 + \Delta B^2} = \sqrt{z+B}$$

Also let $D = A/C$ so

$$\frac{\Delta D}{D} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta C}{C}\right)^2} = \sqrt{\frac{y+B}{(y-B)^2} + \frac{z+B}{(z-B)^2}}$$

Lastly, let $E = \ln D$.

$$\Delta E = \frac{\partial E}{\partial D}\Delta D = \frac{\Delta D}{D}$$

$$\begin{aligned}\frac{\Delta\kappa}{\kappa} &= \sqrt{\left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\Delta l}{l}\right)^2} \\ &= \sqrt{\left(\frac{\Delta D}{ED}\right)^2 + \left(\frac{\Delta l}{l}\right)^2} \\ \Delta\kappa = \kappa &\sqrt{\frac{1}{\ln\left(\frac{y-B}{z-B}\right)^2}\left(\frac{y+B}{(y-B)^2} + \frac{z+B}{(z-B)^2}\right) + \left(\frac{\Delta l}{l}\right)^2}\end{aligned}$$

Using the above error propagation, we calculated $\Delta\kappa$ for each T, and the weighted average for κ using

$$\bar{\kappa} = \frac{\sum_{i=1}^{n} \kappa_i \Delta\kappa_i}{\sum_{i=1}^{n} \Delta\kappa_i}$$

Table 4 shows our calculated values for each T. The weighted average of those values was $k = 0.1909\text{cm}^{-1}$.

T	K
1.000	
0.841	0.1726 ± 0.193
0.559	0.1938 ± 0.074
0.382	0.1927 ± 0.052
0.205	0.1981 ± 0.044
0.170	0.1967 ± 0.044
0.108	0.2022 ± 0.048
0.069	0.2061 ± 0.056
0.043	0.2093 ± 0.069
Weighted Kavg	0.1909 ± 0.0305

Table 4: Our calculated κ values for each T and our final weighted average

Lastly, we measured the density of our aluminum shield to determine if this could be a source for any discrepancy in our measurement for kappa. We measured the height and diameter of each cylindrical block using the vernier caliper, and we measured the mass of each using the triple-beam balance. Table 5 shows our measured values. Our calculated density was 2.739 ± 0.007 g/cm^{-3}, which is only about 1.4% off from the known density of Aluminum (2.70 g/cm^{-3}). The Aluminum barriers are likely an Aluminum alloy, so the 1.4% density fluctuation is not unexpected; however, **this does not account for the 13% variation in our first value for kappa. So we can rule this out as a source of error.** Additional possibilities for the error source are discussed in the post-questions section.

Cylinder number	Al height (cm)	Al diameter (cm)	Al volume (cm^3)	mass (g)	Density (g/cm^3)
1	4.99 ± 0.01	6.37 ± 0.01	159.03 ± 0.48	440 ± 0.1	2.767 ± 0.008
2	3.06 ± 0.01	6.37 ± 0.01	97.52 ± 0.39	264.4 ± 0.1	2.711 ± 0.011
				avg	2.739 ± 0.007

Table 5: Measuring the density of Aluminum

Method	κ (cm^{-1})	N (st devs. from actual)
Linear fit	0.1743 $\pm$ 0.024	2
Weighted average	0.1909 $\pm$ 0.0305	1
Actual	0.201	

Table 6: The results of our measurement of kappa

4 Results

Our results for kappa are summarized in table 6. The value we measured with the linear fit method was within two standard deviations of the actual value, and the value we measure for with the weighted average method was within one standard deviation (within 5%). This suggests that we had some systematic error that caused our values to deviate from the expected value. However, the weighted average method took better account of the errors involved, so we were able to get closer to the expected value. The exact systematic error that caused this discrepancy are explored in the following post-questions section.

5 Post-Questions

1. Is your final measurement of $\kappa_{\text{mean}} \pm \Delta\kappa_{\text{mean}}$ within an error bar or two of the theoretical value 0.201 cm^{-1}?
   The probability of statistical fluctuations (i.e. random chance) yielding a result in error by N standard deviations is roughly e^{-N^2}. Calculate the probability that random chance could lead to the discrepancy between your result and the theoretical value. Does this seem likely?

   The chance that random error accounted for our variation in the linear fit was

   $$e^{-2^2} = 1.83\%$$

   and for the weighted average was

   $$e^{-1^2} = 36.8\%$$

   So, random chance is a source of error for our weighted average calculation, but not for our linear fit. This is likely because the weighted average took better account of the error values of each measurement so we were able to get closer to the actual value of kappa. Therefore, random chance had a higher probability of accounting for the variation in our measurement.

2. Explain any systematic errors in their measurement of κ_{mean} and identify the physical effect which causes the error.

   The largest systematic error is likely the aluminum rod that we used to support the Aluminum shielding. This could cause additional Compton scattering, which would reflect gamma rays back into the detector. The additional gamma rays could be enough to lower the measured value for kappa by the ammount we observed.

3. Study the effects of geometry to see if you can get better agreement with theory.

   The discrepancy in our results are likely due to the geometry of the setup in that we used an Aluminum rail to support the Aluminum shielding masses. This rail could be an additional source of Compton scattering, causing gamma-rays that would otherwise leave the barrier through the bottom to reenter the shielding and make it through to the detector. These additional gamma-rays could be enough to account for the 13% discrepancy in our first value for kappa.

4. Using a fixed measurement time, why do very small and very large values of x_i, the thickness of the aluminum attenuator, produce larger uncertainties $\Delta\kappa_i$ in the attenuation coefficient κ_i?

Our value for $\Delta\kappa$ was

$$\Delta\kappa = \kappa\sqrt{\frac{1}{\ln\left(\frac{y-B}{z-B}\right)^2}\left(\frac{y+B}{(y-B)^2}+\frac{z+B}{(z-B)^2}\right)+\left(\frac{\Delta l}{l}\right)^2}$$

From this formula, we can see that as l gets small, the $\left(\frac{\Delta l}{l}\right)^2$ term is going to blow up, which accounts for the large error for small thicknesses. Also, as l gets large, y (attenuated counts) is going to approach the value of b (background counts). This makes the $\frac{y+B}{(y-B)^2}$ term large, and results in a large error.

6 Conclusion

We showed that gamma-ray intensity decays exponentially with distance inside of a Lead barrier. Although mass per area is an important factor in designing shielding, Lead is a much better shielding material than Aluminum, blocking 3.5% more radiation for every 1 g/cm^2 of mass per area. This effect only gets compounded as you use thicker shielding. Lastly, we measured the transmittance coefficient for Aluminum. With the linear fit method, we achieved a measurement within two standard deviations of the actual value, and the value we measured for with the weighted average method was within one standard deviation. These errors are likely due to the aluminum support, which caused extra gamma rays to reach the detector through additional Compton scattering.