Physics 307 Experiment 6:
Planck Distribution Law and the Stefan-Boltzmann Law

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1 Purpose

To test the Planck distribution law for electromagnetic energy of black body radiation and verify the Stefan-Boltzmann Law of radiation power.

2 Theory

The Planck distribution law determines how much energy an ideal black body will radiate at a given temperature and wavelength.

$$f(\lambda)\ d\lambda = \frac{8\pi hc\lambda^{-5}}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1} d\lambda$$

This relationship is based on the idea that atomic energy levels are quantized, so that a single atom can only inhabit a few specific energy levels (and therefore can only emit photons of those energies). This quantum model perfectly fits the behavior of black body radiation, which classical theories failed to explain. This equation can be integrated to give the Stefan-Boltzmann law

$$P_R/A = \varepsilon\sigma T^4$$

which determines the amount of power radiated by a black body at a given temperature. We will test aspects of both of these models to verify their legitimacy.

3 Pre-questions

1. Using the Planck distribution law, Eq. (1), find the wavelength λ at which f(λ) is a maximum for any temperature T. At $T = 3000K$, for what λ is $f(\lambda)$ a maximum? Hint: When you differentiate $f(\lambda)$ let $a = hc/k_B T$ so that there are fewer terms to track. The result can be written as $5\lambda/a = 1 - \exp(-a/\lambda)$ which must solved either graphically or by iteration. Suggestion: Try $\lambda = a/5$.

$$
\begin{aligned}
f(\lambda) &= \frac{8\pi hc\lambda^{-5}}{\exp\left(\frac{a}{\lambda}\right)-1}\\
\frac{d}{d\lambda}f(\lambda) &= 8\pi hc\Bigg(-5\frac{\lambda^{-6}}{\exp\left(\frac{a}{\lambda}\right)-1}+\frac{a\lambda^{-5}\exp\left(\frac{a}{\lambda}\right)}{\lambda^2(\exp\left(\frac{a}{\lambda}\right)-1)^2}\Bigg)\\
0 &= -5\frac{\lambda^{-6}}{\exp\left(\frac{a}{\lambda}\right)-1}+\frac{a\lambda^{-7}\exp\left(\frac{a}{\lambda}\right)}{(\exp\left(\frac{a}{\lambda}\right)-1)^2}\\
5\lambda\Big(\exp\left(\frac{a}{\lambda}\right)-1\Big) &= a\exp\left(\frac{a}{\lambda}\right)\\
5\lambda\Big(1-\exp\left(\frac{-a}{\lambda}\right)\Big) &= a\\
5\lambda/a &= \Big(1-\exp\left(\frac{-a}{\lambda}\right)\Big)^{-1}
\end{aligned}
$$

Check: $\lambda = a/5$

$$
\begin{aligned}
5(\frac{a}{5})/a &= \Big(1-\exp\left(-5\right)\Big)^{-1}\\
1 &= (0.9933)^{-1}\\
1 &= 1.0067
\end{aligned}
$$

$\lambda = a/5$ is good to around 3 significant figures, which is enough for this experiment. Therefore, the maximum lambda is

$$
\lambda = hc/5k_BT = \frac{2.88\times10^{-03}}{T}
$$

Which for T = 2000 K, $\lambda = 1440$ nm. The metal radiates mostly infrared light.

2. Write an expression that specifies $\varepsilon_{\text{eff}}(T)$ for Equation (6) in terms of $\varepsilon(\lambda, T)$ and a ratio of integrals over all λ of $f(\lambda)\varepsilon(\lambda, T)$ in the numerator and $f(\lambda)$ in the denominator. Note that $\overline{\varepsilon}(T)$ like $\varepsilon(\lambda, T)$ must satisfy $0 \leq \varepsilon(\lambda, T) \leq 1$.

   The energy per volume of a non-ideal black body is

$$
f(\lambda)\varepsilon(\lambda, T)d\lambda
$$

   Since the power ratio is equivalent to the energy ratio, the ε value for equation 6 is just the actual energy radiated divided by the ideal energy.

$$
\frac{f(\lambda)\varepsilon(\lambda, T)\ d\lambda}{f(\lambda)\ d\lambda}
$$

   $\varepsilon_{\text{eff}}(T)$ is just the average value of ε over all lambda, and we can integrate the terms separately to find this.

$$
\varepsilon_{\text{eff}}(T) = \frac{\int f(\lambda)\varepsilon(\lambda, T)\ d\lambda}{\int f(\lambda)\ d\lambda}
$$

3. Discuss the problem with the evaluation of $\bar{\varepsilon}'_{\text{eff}}(T_0)$ in Equation (7). Hint: emissivities are equilibrium quantities and we are using them with a huge difference between T and T_0.

   Because the fillabment is so much hotter than the bulb, the temperature of the bulb T_0 would be very difficult to measure with this setup. Since ϵ_{eff} depends entirely on temperature, we have no way of knowing what the true value is.

4. Is the T_0^4 term in Eq. (7) important at temperatures where the optical pyrometer can be used to measure the temperature ($T \geq 800°$C)? You can assume that the bulb gets no hotter than 300°C.

   No, the T_0 term does not affect the power radiated. Since the filament is around 2.5 times hotter than the bulb, and the temperature is being raised to the fourth power, the filament contributes roughly 2.5^4, or 39 times as much power as the bulb can absorb.

4 Procedure

4.1 Test 1 - Stefan-Boltzmann Law

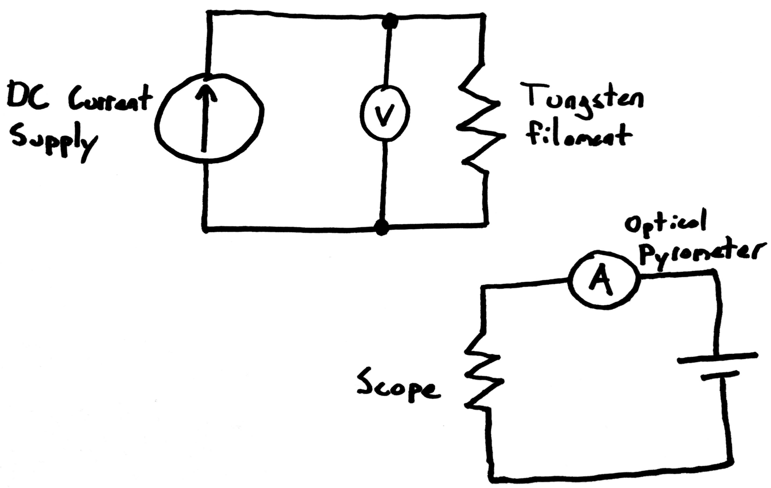

Figure 1: A schematic of the experiment

We split our procedure into two sections. During the first experiment, we wanted to test the T^4 relationship of the Stefan-Boltzmann Law. To do this, we connected a tungsten filament lamp to a current-controlled power supply, and measured the voltage across the lamp using a multimeter. From this we could use $P = IV$ to determine the power radiated from the filament. Then, we used an optical pyrometer to determine the temperature of the filament. We took power and temperature data while adjusting the supplied current in 0.2 A increments from 2.6 to 6.2 A. Figure 1 shows the experimental setup.

Once we gathered our temperature and power data, we used matlab to analyze the relatonship. Since the Stefan Boltzmann Law says the total power radiated per unit area is

$$P_R/A = \epsilon_{\text{eff}}\sigma T^4$$

We can test the exponent value by manipulating the equation with logarithms.

$$\begin{aligned} P_R/\epsilon_{\text{eff}} &= A\sigma T^n \\ \ln(P_R/\epsilon_{\text{eff}}) &= \ln(A\sigma T^n) \\ &= \ln(A\sigma) + \ln(T^n) \\ &= \ln(A\sigma) + n\ln(T) \end{aligned}$$

In this way, we can test the T^4 exponent by plotting $\ln(P_R/\epsilon_{\text{eff}})$ vs $\ln(T^4)$, and the slope of the graph will give us experimental exponent value.

Lastly, we just had to compute the values of ϵ_{eff} for the different temperatures we measured. This accounts for the fact that tungsten is not a perfect black body. We used the model (taken from the NIST datasheet)

$$\epsilon_{\text{eff}} = -0.1073 + 2.528 \times 10^{-4}\ T - 3.518 \times 10^{-8}\ T^2$$

Results are given in the following section.

4.2 Test 2 - Planck distribution law

In our next experiment, we tested the photon flux predicted by the Planck distribution law. The photon flux is dependent on the wavelength of light, so we tested three different wavelengths (red 610.4 nm, green 545.3 nm, and blue 436.5 nm) to see how well the model held. The setup was very similar to Test 1, but this time we also used a monochrometer, notch filter, and a photomultiplier tube (PMT) to measure the electromagnetic energy given off by the filament for a narrow band of wavelengths. We repeated the same procedure, taking data from 2.6 A to 6.2 A, but this time we also measured current from the PMT.

Before we could process the data we had to account for the fact the the tungsten filament is not a perfect black body. To fix this issue, we had to convert all of our temperature values using the equation

$$T_{\text{true}} = \frac{hc/(k_B\lambda_0)}{\ln[\varepsilon_{\text{eff}}(\exp(hc/[k_B T_{\text{obs}}\lambda_0]) - 1) + 1]}$$

5 Results

5.1 Test 1 - Stefan-Boltzmann Law

Figures 2 shows our power vs. temperature data. We fit used a linear fit model only for the high temperature data points, since this is the radiation region we are interested in. Below this region, the log plot deviates from linear because thermal conductivity begins to overpower the radiation produced. We found the slope of this line to be 3.67, which is 8.25% off of the 4.0 slope predicted by the Stefan-Boltzmann Law. The uncertainties of both temperature and power were calculated using the below relationships and are shown on the graph. Since we plotted the log of both functions, relative uncertainties were used and we used the small error approximation of $\Delta\ln(f) \approx 0.434\frac{\Delta f}{f}$.

$$\frac{\Delta P}{P} = \sqrt{\left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}$$

with $\Delta I = \pm 0.05\ A$ and $\Delta V = \pm 0.1\ V$.

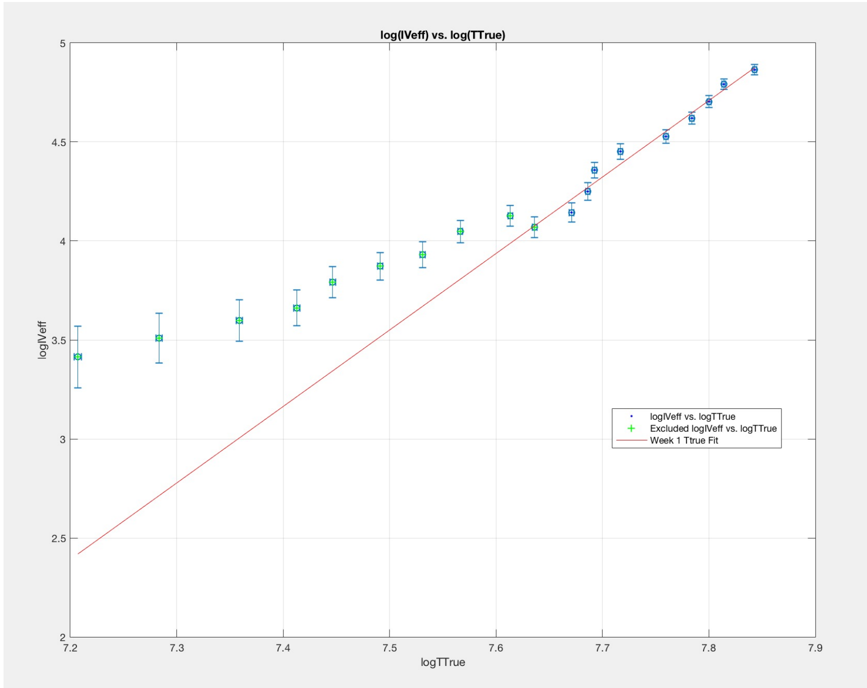

Figure 2: The log plot of Power vs. Temperature.

5.2 Test 2 - Planck distribution law

By plotting $\ln(i)$ vs $\frac{1}{T}$, we can test the photon flux equation (derived from the Planck distribution law). We expect the graph to have slope $\frac{hc}{\lambda k_B}$. From this we can compare our experimental values to the model for each wavelength we tested.

$$v(\lambda, T) = b(\lambda)\varepsilon(\lambda, T)2\pi hc^2\lambda^{-5}\exp\left(-\frac{hc}{\lambda k_B T}\right)d\lambda$$

$$\ln\left(v(\lambda, T)\right) = \ln\left(b(\lambda)\varepsilon(\lambda, T)2\pi hc^2\lambda^{-5}d\lambda\right) - \frac{hc}{\lambda k_B T}$$

$$\ln\left(i(\lambda, T)\right) = C - \frac{hc}{\lambda k_B}\frac{1}{T}$$

Note that since i is proportional to v, the constant of proportionality get absorbed into the y-offset value C, so we should get the same slope if we plot either v or i. Since we used the PMT, we were plotting the current i. Figures 3 to 5 show our results, and the slope values are presented in the following table. Error values were calculated for both $1/T$ and $\ln(T_{\text{eff}})$ and are shown on the graphs.

$$\Delta\frac{1}{T} = T| - 1|\frac{\Delta T}{T} = \Delta T$$

With $\Delta T = 10K$ and $\Delta I = 0.05(I_{\text{fullscale}})$. Again, we used the small error approximation to graph our uncertainties for $\ln(T_{\text{eff}})$.

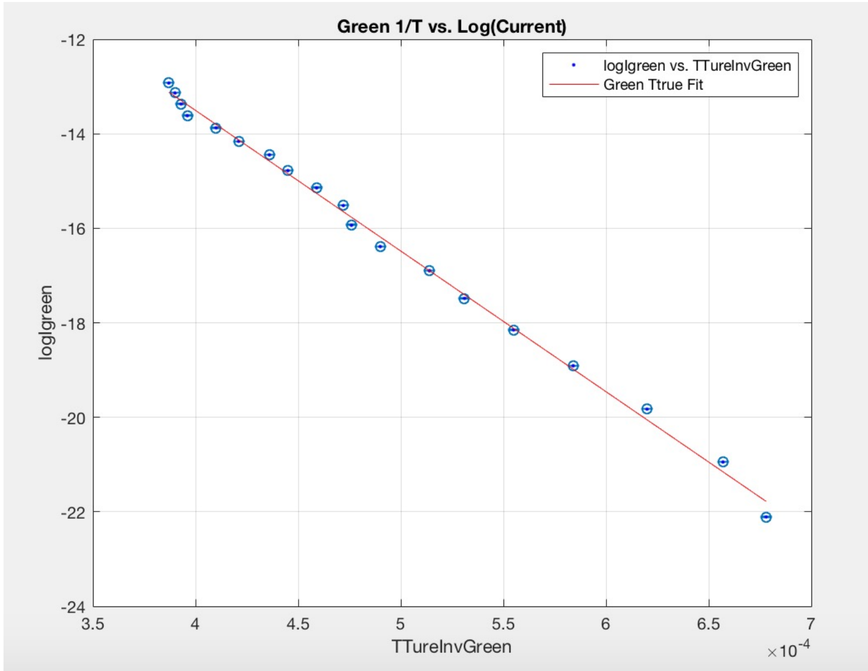

Figure 3: Inverse Temperature vs. Log(Current) for the 610.4 nm wavelength

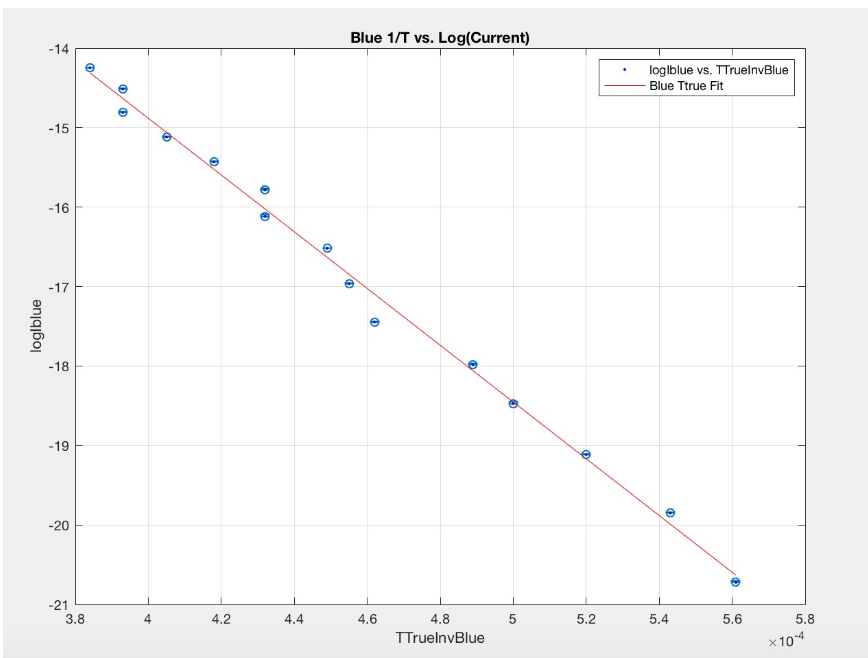

Figure 4: Inverse Temperature vs. Log(Current) for the 545.3 nm wavelength

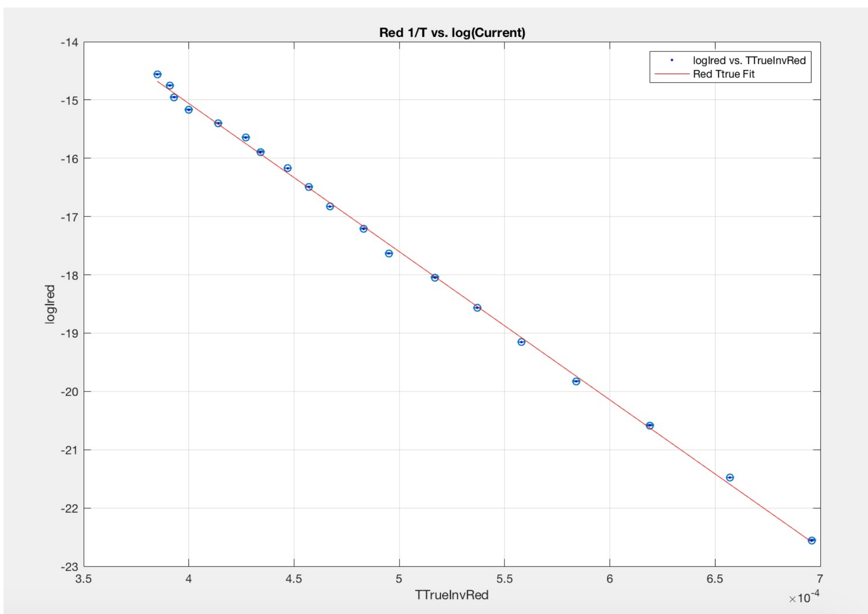

Figure 5: Inverse Temperature vs. Log(Current) for the 436.5 nm wavelength

	Expected ($\times 10^4$)	Measured ($\times 10^4$)	Percent deviation
610.4 nm	-2.357	-2.54	7.17%
545.3 nm	-2.638	-2.98	13.0%
436.5 nm	-3.296	-3.57	8.31%

Comparing our measured slopes with the expected values, we found the results to be fairly close. Interestingly, our data fit the model perfectly (within 0.5%) when we didn't use T_{true}, but after we accounted for the black body variation, the data didn't fit quite as well. Nevertheless, our data shows a strong dependence on the wavelength, as predicted by the Planck distribution.

6 Conclusion

We were able to verify the Planck distribution law and Stefan Boltzmann law each to within approximately 9% of the values predicted by the model. The two models have proven to be very effective at describing block body radiation. In our experiments, the largest source of error was likely the temperature reading from the optical pyrometer. Since the color of the coil needs to be matched by hand, there is a large margin for systematic error. Also, since our values for slope in experiment 2 matched perfectly without adjusting for T_{true}, I'm skeptical that the adjusted temperature data is entirely correct. Future work could include repeating the experiment with multiple people matching the coil to eliminate any systematic error from a single person using the pyrometer.